Lecture 6: Derivatives cont.

September 26, 2016 7:35 PM

<u>Derivatives:</u>

Definition: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ --> "slope of f(x) at x.

<u>Higher derivatives</u>

$f''(x) = (f'(x))'$

$\lim_{h\to 0} \frac{f'(x+h)-f'(x)}{h}$

} second derivative

You can continue to derive a function to a higher and higher derivative, notated by f'(x), f''(x), f'''(x), etc.

Derivatives of polynomials and exponential functions

$\frac{d}{dx}(c) = 0$ if c is a real number

{ alternative notation

constant

$f(x) = x^3 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^3-(x^3)}{h} = \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h} = \lim_{h\to 0}(3x^2 + \underbrace{3xh}_{\to 0} + \underbrace{h^2}_{\to 0}) = 3x^2$

independent of h

$\boxed{\frac{d}{dx}(x^n) = n\cdot x^{n-1}}$ for any real number n!

<u>Example</u>.

1. $f(x) = \frac{1}{x^2}, f'(x) = \left(\frac{1}{x^2}\right)' = (x^{-2})' = -2\cdot x^{-2-1} = \frac{-2}{x^3}$
2. $f(x) = \sqrt{x} = x^{1/2} = {}^{1}/_{2}\, x^{-1/2} = \frac{1}{2\sqrt{x}}$

Constant Multiple Rule

$\frac{d}{dx}(c\cdot f(x)) = c\cdot\frac{d}{dx}f(x)$

Sum Rule

$\frac{d}{dx}(f(x)\pm g(x)) = \underbrace{\frac{d}{dx}f(x)\pm\frac{d}{dx}g(x)}$

if both derivatives exist

<u>Example</u>.

1) $f(x) = 3x^2$
   $f'(x) = (3x^2)' = 3\cdot(x^2)' = 3\cdot 2\cdot x = 6x$

const multiple rule power rule

2) $f(t) = 2t^3 - 5t^2 + 3t + 4$
   $f'(t) = 6t^2 - 10t + 3$

NOTATION: sometimes when we derive for t, then f'(t) is denoted by $\dot{f}(t)$ (for t, time)

Derivatives of exponential functions

$\frac{d}{dx}(e^x) = e^x$, e natural constant

Why?

$f'(x) = (e^x)' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{h\to 0}\frac{e^x\cdot e^h - e^x}{h} = \lim_{h\to 0}\left(\frac{e^x(e^h-1)}{h}\right) = e^x\cdot\lim_{h\to 0}\frac{e^h-1}{h}$

not solvable

e is <u>defined</u> to be the real number satisfying:

e is <u>defined</u> to be the real number satisfying:

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1 \qquad \text{(one definition)}$$

==> $\frac{d}{dx} e^x = e^x$ (using the definition)

<u>Examples</u>:

1) $f(x) = e^x - x^2$
$f'(x) = e^x - 2x$

2) $f(t) = t^{\frac{5}{3}} - 3e^t$
$f'(t) = {}^5/_3\, t^{\frac{2}{3}} - 3e^t$

<u>Product Rule</u>

Recall: We have for sums $(f+g)' = f' + g'$

BUT: $\left(f(x)\cdot g(x)\right)' \neq f'(x)\cdot g'(x)$

ex. $f(x) = x, g(x) = x$

BUT: $f'(x)\cdot g'(x) = (x)'\cdot(x)' = 1\cdot 1 = 1$

<u>Instead</u> we use the **Product Rule**: if f, g are differentiable, then:

$$\left(f(x)\cdot g(x)\right)' = f'(x) * g(x) + f(x) + g'(x)$$

<u>Example</u>:

1) $f(x) = x\cdot e^x$
$f'(x) = x\cdot(e^x)' + (x)'\cdot e^x = x\cdot e^x + 1\cdot e^x = e^x(x+1)$

2) $f(x) = \sqrt{x}\cdot(3+2x)$
$f'(x) = \sqrt{x}\cdot(3+2x)' + (\sqrt{x})'\cdot(3+2x)$
$= \sqrt{x}\cdot 2 + \frac{1}{2\sqrt{x}}(3+2x)$

--> two ways of solving:
1) multiply out and then use rules for polynomials
2) product rule

<u>Quotient Rule</u> (3.2)

again: $\left(\frac{f}{g}\right)' \neq \frac{f'}{g'}$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

<u>Examples to try</u>:

1) $y(x) = \frac{x^4 - 3x^2 + 2}{x^2 + 1}$

$answer\ y'(x) = \frac{2x^5 + 4x^3 - 10x}{(x^2+1)^2}$

2) $f(t) = \frac{e^t + t^3}{t^2\cdot e^t}$

$answer: f'(t) = -\frac{t^4 + t^3 - 2e^t}{t^3\cdot e^t}$